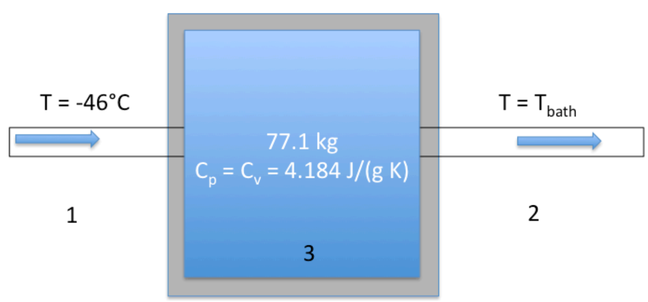
Chemical Engineering Thermodynamics
Quiz 3
January 26, 2017

Last week the high and low temperature in Fairbanks Alaska were -51°F and -49°F (-46.1°C and -45.0°C). Is it possible for a person in a parka (a warm coat) to "freeze to death" just due to breathing this cold air? We will assume that you will "freeze to death" if your **body temperature reaches 30°C**. The body reaches hypothermia at a body temperature of **35.0°C**. (*Assume that the heat production of the body is equal to the heat loss so the body temperature is normally at equilibrium in the absence of breathing.*)

Calculate the time it would take to "freeze to death" (to reach **30°C**) for a well-insulated person (man) under these conditions. (Assume that the person is a bath of water (**77.1 kg**) loosing heat to an air stream with an average air flow rate of **620 liter/min**, $\mathbf{T_{in} = -46°C}$, $\mathbf{T_{out} = T_{bath}}$, and that the initial water bath temperature is 98.6°F (**37.0 °C**). Ignore body heat production and heat transfer to the environment. Also calculate the time to reach a state of hypothermia, **35.0°C**. Assess the importance of ignoring heat production by the body, 2,500 cal/day (**7.26 kJ/min**).

1000 L = 1 m^3.
Air density is 1.46897 kg/m^3 at -45.5C and 1.007 bar.
Assume $C_{pair} = 7/2\ R$ and $C_{vair} = 5/2\ R$.
The molecular weight of air is 29 g/mole.
For water $C_p = C_v = 4.184$ J/(g K).
R = 8.314 J/(K mole).



1) First write an expression for dQ from the air stream with T_{bath}, (dm_{air}/dt) and dt as variables.
2) Then write an expression for d(m_{bath} U) as a function of dT_{bath}.
3) Equate the two and collect the T_{bath} terms on the left and time on the right.
4) Integrate this expression to get an expression for T_{bath} as a function of time.
5) Set T_{bath} to 30°C to find the time till "freezing to death". (Also find for 35°C for hypothermia.)
6) Compare the added dQ (7.26 kJ/min) dt with the maximum loss to air dQ from your expression in item 1 and make an assessment of the impact on the resulting time.

Answers Quiz 3
ChE Thermo
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1) Air Stream

$$d\underline{Q} = C_p \,\Delta T \left(\frac{dm_{air}}{dt}\right) dt$$

$$d\underline{Q} = \frac{7}{2} R \left(T_{bath}^{\,°C} + 46\,°C\right) 31.4 \ \mathrm{mol/min}$$

$$\frac{dm_{air}}{dt} = 620 \frac{\ell}{min} \ \frac{1\,m^3}{1000\,\ell} \ \frac{1.47 \frac{kg}{m^3}}{29\, g/mol} \ \frac{kg}{1000\, g}$$

$$\frac{dm_{air}}{dt} = 31.4 \ \frac{mole}{min}$$

$$d\underline{Q} = 29.1 \frac{J}{°K\,mol} \ 31.4 \frac{mole}{min} \left(T_{bath}^{\,°C} + 46\,°C\right)$$

$$\boxed{d\underline{Q} = -913 \ \frac{J}{°K\,min} \left(T_{bath}^{\,°C} + 46\,°C\right)}$$

2)

$$d\underline{Q} = -d\left(m_{bath}\,\Delta u\right)$$

$$= -\, m_{bath}\, C_{V_{wat}}\, dT_{bath}$$

$$= -77{,}100\,g \ 4.18 \frac{J}{g\,°K} \ dT_{bath}$$

$$\boxed{d\underline{Q} = 322{,}000 \ \frac{J}{K} \ dT_{bath}}$$

3)

$$\frac{+\,dT_{bath}}{\left(T_{bath}^{\,°C} + 46\,°C\right)} = \frac{-913 \frac{J}{K\,min}}{322{,}000 \frac{J}{K}} \ dt$$

$$\boxed{\frac{dT_{bath}}{\left(T_{bath}^{\,°C} + 46\,°C\right)} = -2.84\,e{-3}\left(\frac{1}{min}\right) dt}$$

4)
$$\int_{37^\circ C}^{T_{bath\,final}\,^\circ C} \frac{+dT_{bath}}{(T_{bath}\,^\circ C + 46^\circ C)} = \ln\left(\frac{T_{bath\,final}\,^\circ C + 46^\circ C}{83^\circ C}\right)$$

$$x = (T_{bath}\,^\circ C + 46^\circ C)$$
$$dx = dT_{bath}$$

$$\ln\left(\frac{T_{bath\,final} + 46^\circ C}{83^\circ K}\right) = -2.84e^{-3}\,(min)^{-1}\,t$$

$$\boxed{t = -352\,min\,\ln\left(\frac{T_{bath\,final}\,^\circ C + 46^\circ C}{83^\circ K}\right)}$$

5) For $T_{bath\,final} = 30^\circ C$

$$t_{30^\circ C} = -352\,min\,\ln\left(\frac{(30+46)^\circ K}{83^\circ K}\right) = 31.0\,min$$

$$t_{35^\circ C} = -352\,min\,\ln\left(\frac{(35+46)^\circ K}{83^\circ K}\right) = 8.60\,min$$

6) Take the maximum $\Delta T = 83^\circ K$

$$dQ = -914\,\frac{J}{K\,min}\,(83^\circ K)\,dt = -75.9\,KJ/min\;dt$$

we add $7.26\,KJ/min$ ~~…~~

Food calories are about 10%
of the heat lost by
breathing. You would
still have problems.
The only thing keeping you alive
in Alaska is the slow heat
transfer in the body.